DIFFERENTIAL EQUATION

EXERCISE – I HINTS & SOLUTIONS

Sol.1 D

$$\left(1+3\frac{dy}{dx}\right)^{2/3} = 4\ \frac{d^3y}{dx^3}$$

$$\left(1+\frac{3dy}{dx}\right)^{2} = \left(4\frac{d^3y}{dx^3}\right)^3$$

order = 3
Degree = 3

Sol.2 B

$y^2 = 4ax + k$

$$2y\ \frac{dy}{dx} = 4a$$

$$2y\ \frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = 0$$

degree = 1
order = 2

Sol.3 C

$$\left(\frac{dy}{dx}\right)^{1/3} = 4\frac{d^2y}{dx^2} + 7x$$

$$\left(\frac{dy}{dx}\right) = \left(4\frac{d^2y}{dx^2}+7x\right)^3$$

order =2 = a
degree = 3 = b
a + b = 5

Sol.4 C

$y = e^{mx}$
$D^3y - 3D^2y - 4\ Dy + 12\ y = 0$
$m^3\ e^{mx} - 3m^2\ e^{mx} - 4me^{mx} + 12\ e^{mx} = 0$
$m^3 - 3m^2 - 4m + 12 = 0$
$m^2\ (m - 3) - 4\ (m - 3) = 0$
$m = 3, 2, -2$
Two Natural numnber of m possible

Sol.5 D

$y = mx + c$
$y' = m$
$D^2y - 3Dy - 4y = -4x$
$0 - 3m - 4\ (mx + c) = -4x$
$-3m - 4mx - 4C = -4x$
$-4m = -4 \quad \Rightarrow m = 1$

$$-3m - 4C = 0 \ \Rightarrow\ 4C = -3m \ \Rightarrow\ C = -\frac{3}{4}$$

Sol.6 B

$y = a + bx + ce^{-x}$
$y' = b - ce^{-x}$
$y'' = ce^{-x}$
$y''' = -ce^{-x}$
$y''' = -y'' \quad \Rightarrow \quad y''' + y'' = 0$

Sol.7 B

$Ax^2 + By^2 = 1$
$2Ax + 2Byy' = 0$

$$Ax + Byy' = 0 \quad \Rightarrow \quad \frac{A}{B} = \frac{-yy'}{x}$$

$A + B\ (y')^2 + Byy'' = 0$

$$\frac{A}{B} + \left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2} = 0$$

$$\frac{-y}{x}\ \frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2 + y\ \frac{d^2y}{dx^2} = 0$$

Order = 2
Degree = 1

Sol.8 A

$(x - h)^2 + (y - k)^2 = a^2$

$$(x - h) + (y - k)\ y' = 0 \quad \Rightarrow \quad y' = \frac{-(x-h)}{(y-k)}$$

$$1 + (y - k)\ y'' + (y)^2 = 0 \quad \Rightarrow \quad y'' = \frac{-a^2}{(y+k)^3}$$

(A) option satisfy the given conditions

Sol.9 C

$y = e^{(k + 1)x}$
$y' = (k + 1)e^{(k + 1)x}$
$y'' = (k + 1)^2\ e^{(k + 1)x}$

$$\frac{d^2y}{dx^2} - 4\ \frac{dy}{dx} + 4y = 0$$

$(k + 1)^2 - 4\ (k + 1) + 4 = 0$
$k^2 + 2k + 1 - 4k = 0$
$(k - 1)^2 = 0$
$k = 1$

Sol.10 A

$$x^2 + y^2 - 2ay = 0 \Rightarrow a = \frac{(x^2+y^2)}{2y}$$

$2x + 2yy' - 2ay' = 0$

$$x + yy' - \left(\frac{x^2+y^2}{2y}\right)y' = 0$$

$$x + y'\left(\frac{y^2-x^2}{2y}\right) = 0$$

$2xy + y'(y^2 - x^2) = 0$
$y'(x^2 - y^2) = 2xy$

Sol.11 B
$ydy = (1 - x)\,dx$

$$\frac{y^2}{2} = x - \frac{x^2}{2} + C$$

$x^2 + y^2 - 2x - C = 0$

Sol.12 A
$y\ell ny + xy' = 0$

$$y\ell ny + x\frac{dy}{dx} = 0 \qquad \Rightarrow \frac{dx}{x} + \frac{dy}{y\ell ny} = 0$$

$\ell nx + \ell n(\ell ny) = \ell nC$
$x(\ell ny) = C$
$y(1) = e$
$\ell ne = C \Rightarrow C = 1 \qquad x(\ell ny) = 1$

Sol.13 D

$$\int_0^x t\,y(t)\,dt = x^2 y(x)$$

Using leibnitz
$xy = 2xy + x^2y'$
$y = 2y + xy'$
$xy' + y = 0$

$$x\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{y} + \frac{dx}{x} = 0$$

$\ell n\, xy = C$
$xy = k\ ;\ (2, 3) \Rightarrow k = 0$
$xy = 6$

Sol.14 A

$$\frac{dy}{dx} = \sin(10x + 6y)$$

Put $10x + 6y = t \Rightarrow 10 + 6\frac{dy}{dx} = \frac{dt}{dx}$

$$\frac{dt}{dx} - 10 = 6\sin t \Rightarrow \frac{dt}{10+6\sin t} = dx$$

$$\frac{dt}{10+6\left(\dfrac{2\tan\frac{t}{2}}{1+\tan^2\frac{t}{2}}\right)} = dx$$

$$\frac{\sec^2\frac{t}{2}\,dt}{10+10\tan^2\frac{t}{2}+12\tan\frac{t}{2}} = dx$$

put $\tan\frac{t}{2} = z \Rightarrow \sec^2\frac{t}{2}\,dt = 2dz$

$$\frac{2dz}{10(1+z^2)+12z} = dx$$

$$\frac{dz}{5z^2+6z+5} = dx \Rightarrow \frac{dz}{\left(z+\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2} = 5\,dx$$

$$\frac{5}{4}\tan^{-1}\frac{z+\frac{3}{5}}{\frac{4}{5}} = 5x + 5k$$

$$\tan^{-1}\frac{5z+3}{4} = 4x + C$$

$$(0, 0) \Rightarrow C = \tan^{-1}\frac{3}{4}$$

$$\tan^{-1}\frac{5z+3}{4} - \tan^{-1}\frac{3}{4} = 4x$$

$$\frac{\frac{5z+3}{4}-\frac{3}{4}}{1+\left(\frac{5z+3}{4}\right)\frac{3}{4}} = \tan 4x \Rightarrow \frac{20z}{25+15z} = \tan 4x$$

$4z = (5 + 3z)\tan 4x$
$z(4 - 3\tan 4x) = 5\tan 4x$

$$z = \frac{5\tan 4x}{4-3\tan 4x} \Rightarrow \tan\frac{t}{2} = \left(\frac{5\tan 4x}{4-3\tan 4x}\right)$$

$$5x + 3y = \tan^{-1}\left(\frac{5\tan 4x}{4-3\tan 4x}\right)$$

$$y = \frac{1}{3}\tan^{-1}\left(\frac{5\tan 4x}{4-3\tan 4x}\right) - \frac{5x}{3}$$

Sol.15 B

$$\frac{dy}{dx} = \frac{y}{x}\left(\ell n\left(\frac{y}{x}\right)+1\right)$$

Put $y = tx \Rightarrow \frac{dy}{dx} = t + x\frac{dt}{dx}$

$t + x\frac{dt}{dx} = t(\ell nt + 1)$

$x\frac{dt}{dx} = t\,\ell n\,t$

$\frac{dt}{t\ell nt} - \frac{dx}{x} = 0$

$\ell n(\ell n\,t) - \ell n\,x = C$

$\frac{\ell nt}{x} = k$

$\ell nt = kx$

$\ell n\left(\frac{y}{x}\right) = kx$

Sol.16 A

$$\frac{dy}{dx} = \frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$$

Put $y = tx \Rightarrow \frac{dy}{dx} = t + x\frac{dt}{dx}$

$t + x\frac{dt}{dx} = t - \cos^2(t) \Rightarrow x\frac{dt}{dx} = -\cos^2 t$

$\sec^2 t\,dt + \frac{dx}{x} = 0$

$\tan t + \ell n\,x = C \Rightarrow \tan t = C - \ell n\,x$

$\tan\frac{y}{x} = C - \ell n\,x \qquad \left(1, \frac{\pi}{4}\right)$

$C = 1$

$\tan\frac{y}{x} = 1 - \ell n\,x \Rightarrow \tan\frac{y}{x} = \ell n\frac{e}{x}$

$y = x\tan^{-1}\left(\ell n\frac{e}{x}\right)$

Sol.17 C

$(2x - 10y^3)\frac{dy}{dx} + y = 0$

$\frac{dy}{dx} = \frac{y}{10y^3 - 2x} \Rightarrow \frac{dx}{dy} = \frac{10y^3 - 2x}{y}$

$\frac{dx}{dy} + \frac{2x}{y} = 10y^2$

I.F. $= e^{\int\frac{2}{y}dy} = y^2$

$x(y^2) = \int 10y^4 dy$

$y^2x = 2y^5 + C$

Sol.18 B

$(1 + y^2)\,dx + (x - e^{\tan^{-1}y})\,dy = 0$

$dx + \frac{(x - e^{\tan^{-1}y})dy}{(1+y^2)} = 0$

put $e^{\tan^{-1}y} = t \Rightarrow \frac{e^{\tan^{-1}y}}{(1+y^2)}dy = dt$

$\frac{dy}{(1+y^2)} = \frac{dt}{t}$

$dx + (x - t)\frac{dt}{t} = 0$

$tdx + xdt - tdt = 0$

$d(xt) - tdt = 0$

$xt = \frac{t^2}{2} + C$

$xe^{\tan^{-1}y} = \frac{1}{2}e^{2\tan^{-1}y} + C$

Sol.19 A

$y' + y\phi' - \phi\phi' = 0$

$y' + \phi'(y - \phi) = 0$

$dy + \phi'(y - \phi)\,dx = 0$

Let $\phi = t \qquad \Rightarrow \phi'\,dx = dt$

$dy + (y - t)\,dt = 0$

$\frac{dy}{dt} + y = t$

I.F. $= e^t$

$ye^t = \int te^t\,dt$

$ye^t = te^t - e^t + C$

$y = t - 1 + ce^{-t}$

$y = \phi(x) - 1 + ce^{-\phi(x)}$

Sol.20 B

$e^x(x + 1)\,dx + (ye^y - xe^x)\,dy = 0$

Put $\quad xe^x = t \quad \Rightarrow \quad e^x(x + 1)\,dx = dt$

$dt + (ye^y - t)\,dy = 0$

$$\frac{dy}{dt} = \frac{1}{t - ye^y} \Rightarrow \frac{dt}{dy} = t - ey^y$$

$$\frac{dt}{dy} - t = -ye^y$$

I.F. = e^{-y}

$$(xe^x)(e^{-y}) = \frac{-y^2}{2} + C\ ; \quad (0, 0) \Rightarrow C = 0$$

$$(xe^x)(e^{-y}) = \frac{-y^2}{2}$$

$2xe^x + y^2e^y = 0$

Sol.21 D

$ydx + xdy + x\,(xy)\,dy = 0$

Let $xy = t \qquad \Rightarrow \quad x = \dfrac{t}{y}$

$xdy + ydx = dt$

$$dt + \left(\frac{t}{y}\right) t\, dy = 0$$

$$\frac{dt}{t^2} + \frac{dy}{y} = 0 \Rightarrow -\frac{1}{t} + \ell ny = C$$

$$\frac{-1}{xy} + \ell ny = C$$

Sol.22 B

$$y^5x + y - x\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} - y - y^5 x = 0$$

$$\frac{dy}{dx} - \frac{y}{x} = y^5 \qquad \text{(LDE)}$$

$$\frac{1}{y^5}\frac{dy}{dx} - \frac{1}{y^4x} = 1$$

$$\frac{1}{y^4} = t \qquad \Rightarrow \frac{1}{y^5}\frac{dy}{dx} = -\frac{1}{4}\frac{dt}{dx}$$

$$-\frac{1}{4}\frac{dt}{dx} - \frac{t}{x} = 1$$

$$\frac{dt}{dx} + \frac{4t}{x} = -4 \qquad \text{(LDE)}$$

$$\text{I.F.} = e^{\int \frac{4}{x}dx} = x^4$$

$$x^4 . t = -\frac{4x^5}{5} + C$$

$$\frac{x^4}{y^4} = -\frac{4x^5}{5} + C$$

$$\frac{x^5}{5} + \frac{1}{4}\left(\frac{x}{y}\right)^4 = k$$

Sol.23 C

$$\frac{xdy}{x^2+y^2} = \frac{ydx}{x^2+y^2} - dx$$

$$\frac{xdy - ydx}{x^2+y^2} = -dx$$

$$\frac{\frac{xdy - ydx}{x^2}}{1+\left(\frac{y}{x}\right)^2} = -dx \Rightarrow \frac{d(y/x)}{1+(y/x)^2} = -dx$$

$$d\left(\tan^{-1}\frac{y}{x}\right) = -dx \Rightarrow \tan^{-1}\frac{y}{x} = -x + C$$

$$\frac{y}{x} = \tan(C - x) \Rightarrow \qquad y = x\tan(C - x)$$

Sol.24 B

$$\frac{dy}{dx} = 100 - y$$

$-\ell n\,(100 - y) = x + C\ ; \quad y(0) = 50$

$-\ell n\,(100 - y) = x - \ell n\, 50 \Rightarrow \quad C = -\ell n\, 50$

$$\ell n\left(\frac{100-y}{50}\right) = -x$$

$100 - y = 50\, e^{-x}$

$y = 100 - 50\, e^{-x}$